Mathematics C Assignment

Structures and Patterns

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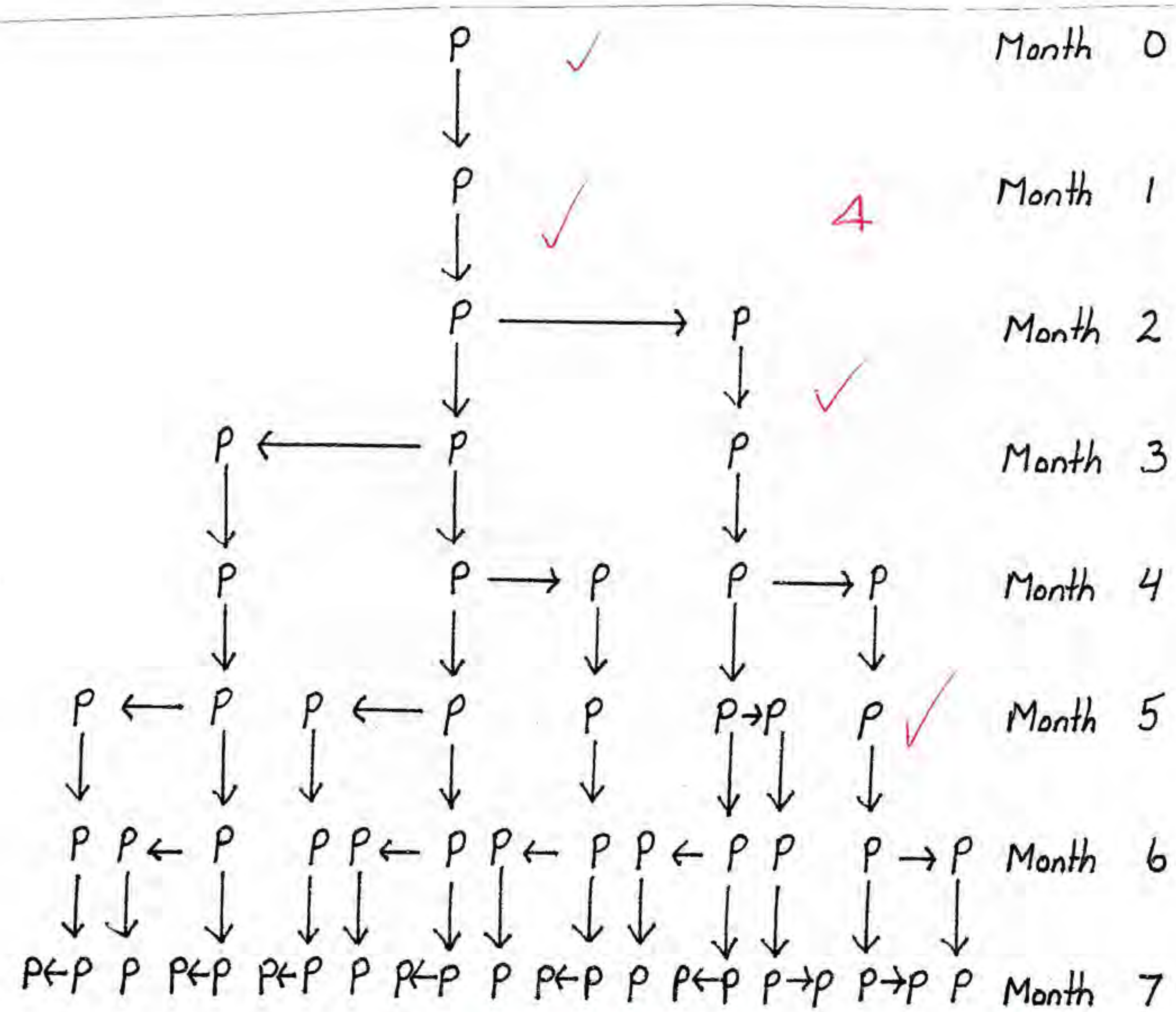
27 October, 1995.

THE FIBONACCI SEQUENCE

Leonardo Fibonacci, also known as Leonardo de Pisa, was an Italian mathematician of the late 12th and early 13th centuries. He advocated the adoption of Arabic notation and extended the material then known in geometry and trigonometry.

(a) A pair of rabbits one month old are too young to produce more rabbits, but suppose that in their second month and every month thereafter they produce a new pair. If each new pair of rabbits does the same, and none of the rabbits die, how many pairs of rabbits will there be at the beginning of each month? Describe your results.

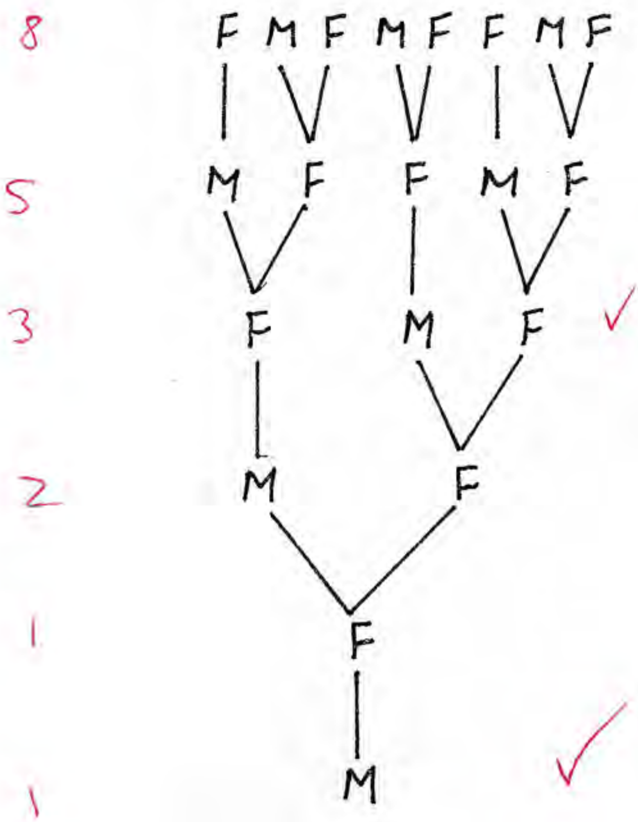
At the beginning of month 1, there will be 1 pair of rabbits.
At the beginning of month 2, there will be 1 pair of rabbits.
At the beginning of month 3, there will be 2 pairs of rabbits.
At the beginning of month 4, there will be 3 pairs of rabbits.
At the beginning of month 5, there will be 5 pairs of rabbits, etc.



may form fibonacci sequence ✓ 1½

5½/6

(b) Now apply this problem to that of the family tree of a male bee. A male bee has only one parent, his mother, while a female bee has both father and mother. The family tree of a male bee has a strange pattern as a result. If the male is represented by the symbol M and each female by the symbol F, what would the family tree look like?



should also note pattern -ie Fibonacci

5/6

(c) A great 18th century French mathematician named Joseph Lagrange did not become interested in Mathematics until he was 17. He discovered a pattern in the remainders formed by dividing each term in the Fibonacci sequence by 4. The remainders for the first four terms are the terms themselves. Find the remainders for the rest of the terms in this sequence. For example, the ninth term, 34, gives the remainder of 2 when divided by 4. What is the pattern in the sequence of the remainders?

Fibonacci seq.: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 114, ...

Divide terms by 4.

Remainders: 1, 1, 2, 3, 1, 0, 1, 1, 2, 3, 1, 0, ...

n	1	2	3	4	5	6	7	8	9	10	11	12
Fibonacci	1	1	2	3	5	8	13	21	34	55	89	144
Remainder	1	1	2	3	1	0	1	1	2	3	1	0

The pattern in the sequence of remainders is that of 1, 1, 2, 3, 1, 0.

3/3

(d) Find the square of the first eight terms of the Fibonacci sequence. Then add each pair of consecutive squares to make a new sequence. What do you notice?

n	1	2	3	4	5	6	7	8
Fibonacci	1	1	2	3	5	8	13	21
Fib. sq.	1	1	4	9	25	64	169	441

Add each pair of consecutive squares to make a new sequence.

$1 + 1 = 2$
$1 + 4 = 5$
$4 + 9 = 13$
$9 + 25 = 34$
$25 + 64 = 89$
$64 + 169 = 233$
$169 + 441 = 610$

New sequence becomes 2, 5, 13, 34, 89, 233, 610, ...
Compare new sequence to original Fibonacci sequence.

New seq.			2		5		14		34		89
Fibonacci seq.	1	1	2	3	5	8	13	21	34	55	89
			✓		✓		✓		✓		✓

Clearly, the terms of the new sequence formed by adding the consecutive squares of the Fibonacci sequence correspond to every second term of the original Fibonacci sequence.

3/3

(e) The pattern below is also based on a sequence of squares in the Fibonacci series. Write the next two lines of the pattern and check to see if they are correct.

Pattern based on a sequence of squares in the Fibonacci series.

$1^2 + 1^2 = 1.2$
$1^2 + 1^2 + 2^2 = 2.3$
$1^2 + 1^2 + 2^2 + 3^2 = 3.8$
$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 5.8$
$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = 8.13 = 48$ (correct!)
$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2 = 13.21 = 273$ (correct!)

Now write in the missing numbers in the pattern below. This pattern is based on cubes of consecutive terms of the Fibonacci sequence. What do you notice about the numbers on the right?

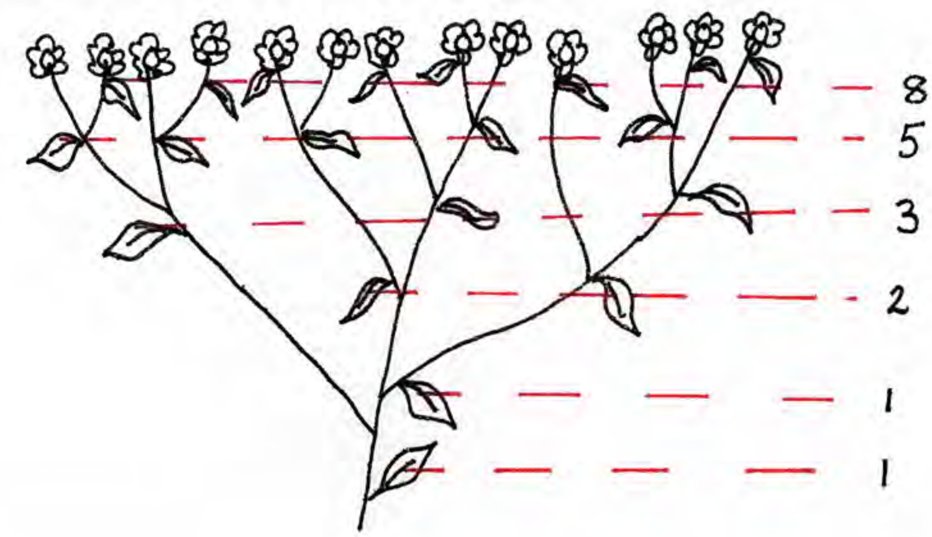
Pattern based on cubes of consecutive terms in the Fibonacci sequence.

$2^3 + 3^3 - 1^3 = 34$
$3^3 + 5^3 - 2^3 = 144$
$5^3 + 8^3 - 3^3 = 610$

9th, 12th, 15th Terms

The numbers on the right correspond to every third term of the Fibonacci Sequence. $3^{1/2}/4$

(f) An area of biology in which Fibonacci numbers appear is plant growth; the numbers of peals of many flowers are Fibonacci numbers. Fibonacci numbers also appear in the arrangement of leaves on the stems of plants. What Fibonacci numbers can you find in the drawing of this plant?

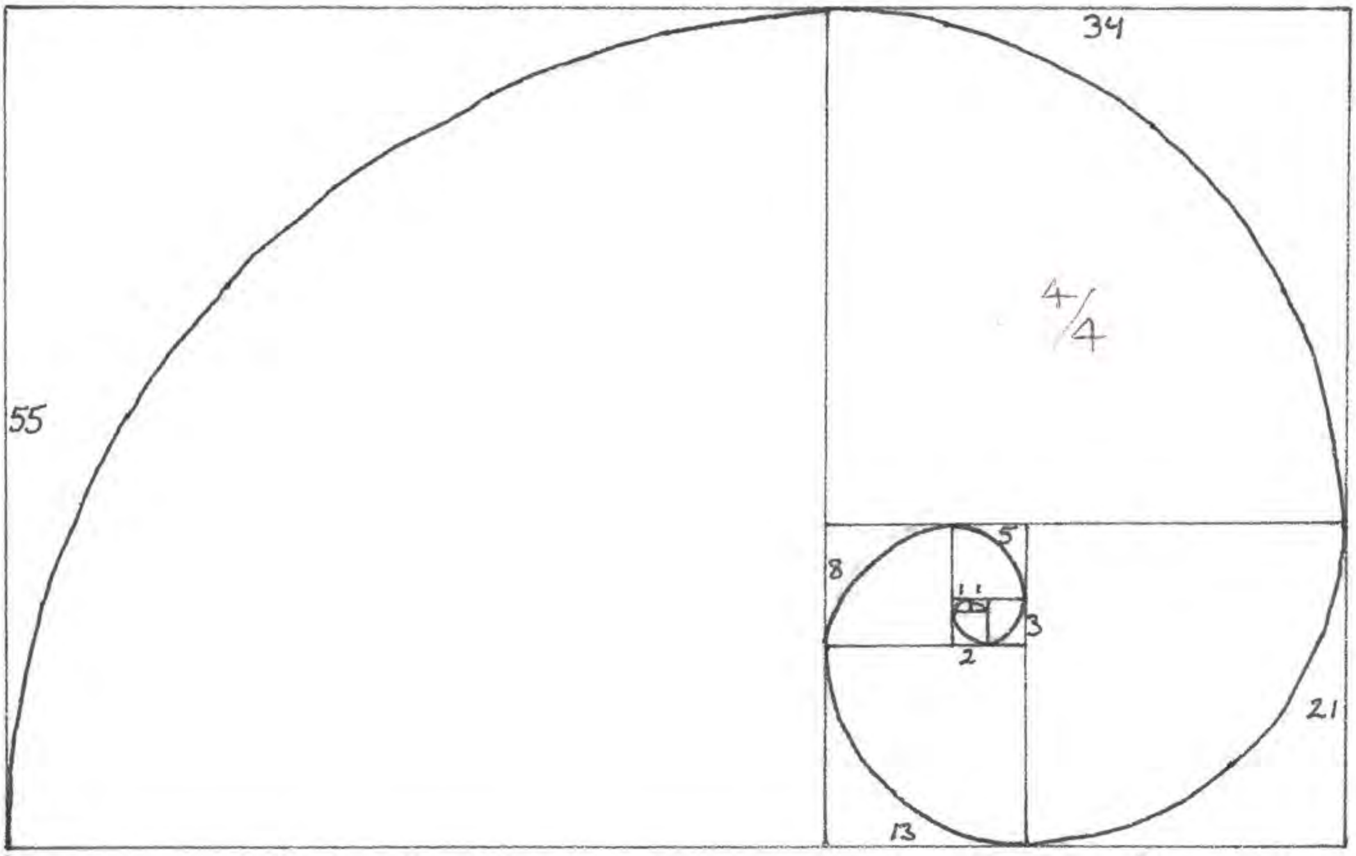


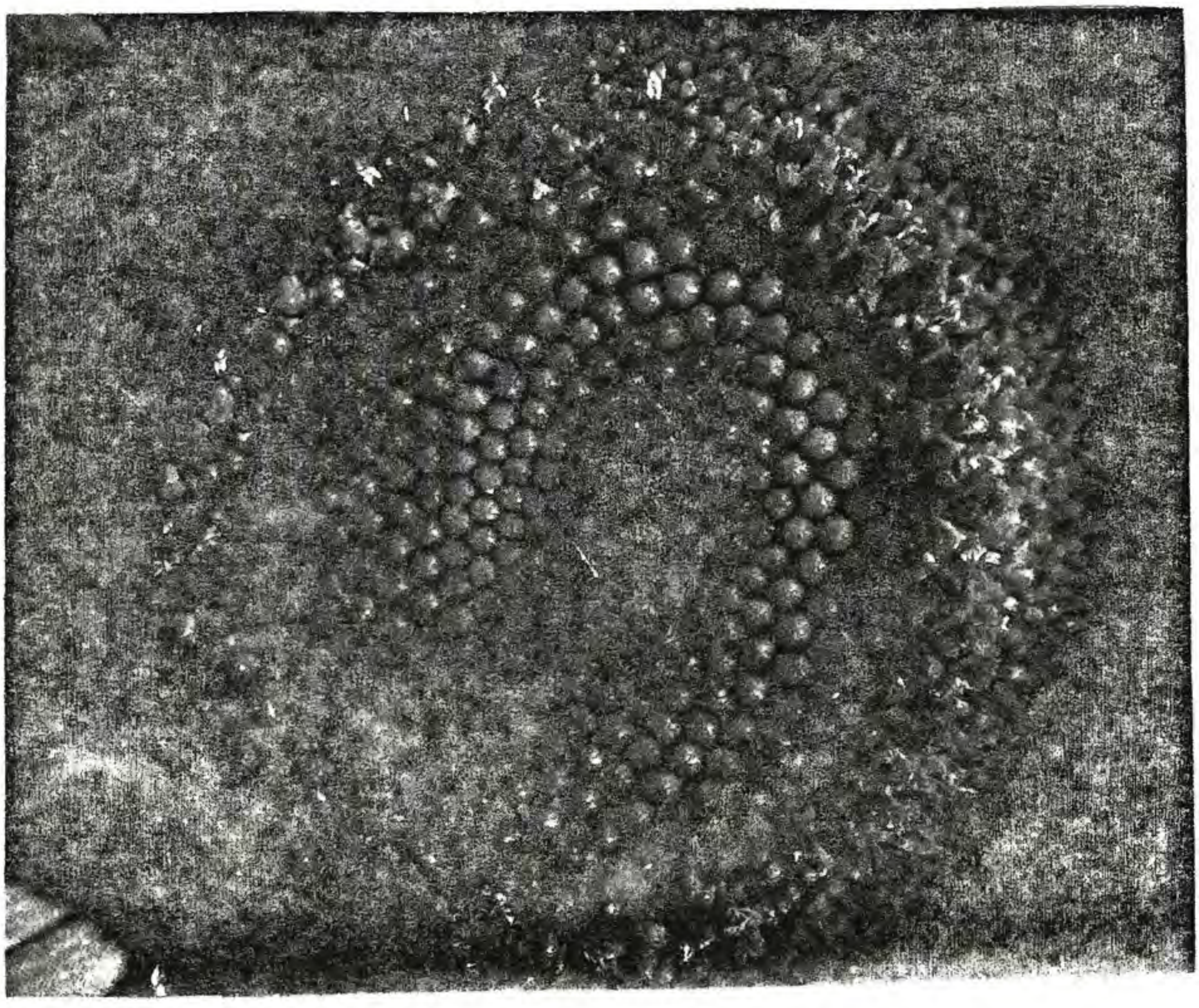
✓

3/3

(g) The Fibonacci sequence is also closely related to the Logarithmic Spiral. Use graph paper to construct a diagram similar to the one shown below. The resulting curve, based on the Fibonacci sequence, is close to the shape of the chambered Nautilus!!

The Logarithmic Spiral





PASCALS TRIANGLE

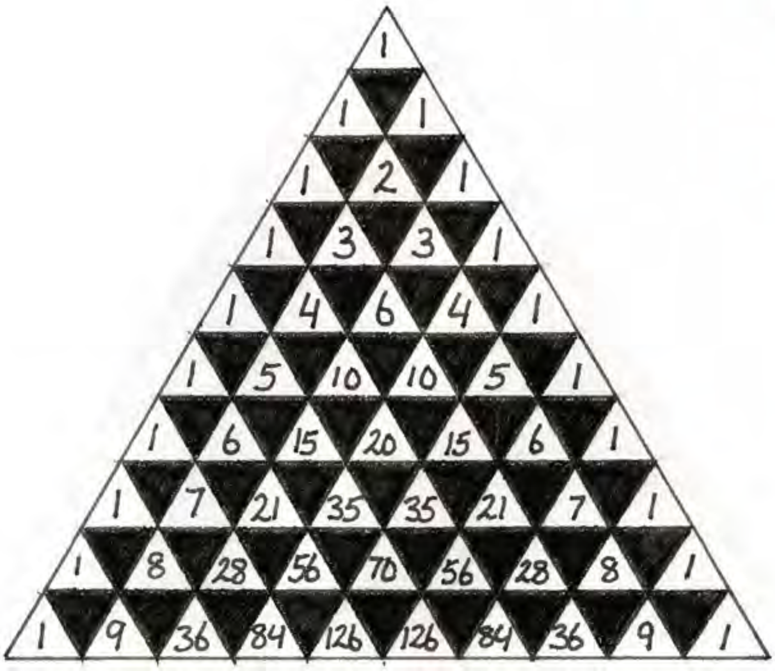
Blaise Pascal (1623-1662), was a French religious philosopher, mathematician and scientist.

Pascal was born in Clermont-Ferrand. His father, a civil servant, took charge of his education and taught him only those subjects which he wanted him to know. These were mostly ancient languages, and Pascal's father refused to teach him any of the sciences until he found that at the age of twelve, the boy had discovered geometry and taught it to himself. Before he was sixteen, he wrote a paper on conic sections which won the respect of the mathematicians of Paris; at nineteen he invented a calculating machine. In addition, Pascal discovered the properties of the cycloid, contributed to the advance of differential calculus and is credited with founding the modern theory of probability, including the mathematical triangle now known as Pascal's Triangle.

(a) In a rather unusual family, all 8 of the children are girls. Since the probability of a child being a boy or girl are about the same, it would seem as if, in a family of this size, 4 boys and 4 girls would be much more likely. Exactly how do the probabilities of a family having 8 girls and a family having 4 girls and 4 boys compare?

2/2

Pascal's Triangle is a ready reference for finding the odds governing combinations. Each number within the triangle is obtained by adding the two numbers immediately right and left above it. Computations are always made horizontally by rows: there are eight row shown, covering the chances ruling groups of one to eight. The sum of the numbers in any row gives the total arrangements of combinations possible within that group.



Rows
0
1
2
3
4
5
6
7
8
9

1½/2

Therefore, to determine the probability of any given boy-girl combination in a family of eight children, the numbers of the eighth row are first added which gives a total of 256. The triangles at the ends of the row stand for the chances for the least likely combinations - that is, all boys or all girls: 1 in 256. The center triangle - 70 - applies to four boys, four girls, for which the chances are 70 in 256. 5½/6

(b) Represent the number of ways in which 3 coins can turn up, then 4 coins. Now depict how 8 coins can turn up and relate this back to the original problem regarding 8 girls in a family.

The third row of the triangle (1, 3, 3, 1) represents the way in which 3 coins can turn up when they are tossed.

ie.					
	Number of Heads	3	2	1	0
	Number of Ways	1	3	3	1
	Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

✓

The fourth row of the triangle (1, 4, 6, 4, 1) represents the way in which 4 coins can turn up when they are tossed.

ie.						
	Number of Heads	4	3	2	1	0
	Number of Ways	1	4	6	4	1
	Probability	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

✓

The eighth row of the triangle (1, 8, 28, 56, 70, 56, 28, 8, 1) represents the way in which 8 coins can turn up when they are tossed.

ie.										
	Number of Heads	8	7	6	5	4	3	2	1	0
	Number of Ways	1	8	28	56	70	56	28	8	1
	Probability	$\frac{1}{256}$	$\frac{8}{256}$	$\frac{28}{256}$	$\frac{56}{256}$	$\frac{70}{256}$	$\frac{56}{256}$	$\frac{28}{256}$	$\frac{8}{256}$	$\frac{1}{256}$

Once again, the probability of a rare sequence such as repeated heads or repeated tails in coins when they are tossed can be calculated using Pascal's Triangle.

The sum of the numbers in any row gives the total arrangements of combinations possible within that group. Using both examples (a) and (b), to determine the probability of any given boy-girl combination in a family of eight children or any given head-tail combination in eight coins when they are tossed, the numbers in the eighth row are first added, which gives a total of 256. The triangles at the ends of the row stand for the chances for the least likely combinations - that is, all boys or all girls, or all heads or all tails: 1 in 256. The second triangles from the ends apply to the next most likely combinations (seven boys, one girl or vice versa; or seven heads, one tail or vice versa) - 8 in 256. The third triangles from the ends apply to the next most likely combinations (six boys, two girls or vice versa; or six heads, two tails or vice versa) - 28 in 256. The center number - 70 - applies to four boys, four girls, or four heads, four tails, for which the chances are 70 in 256.

GOOD

5/5

(c) By multiplying, find the values of 11^2, 11^3 and 11^4. What do these numbers have to do with Pascal's Triangle?

$11^2 = 11 * 11 = 121$
$11^3 = 11 * 11 * 11 = 1331$
$11^4 = 11 * 11 * 11 * 11 = 14641$

- ie - 2nd, 3rd & 4th Rows of Pascal's Triangle

11^n corresponds to the nth row of Pascal's triangle, where n is an integer and $2 < n < 4$.

$2\frac{1}{2}/3$

(d) Sum the numbers of Row 1, then Row 2, then Row 3, etc. The resulting numbers are part of a simple number sequence type. What kind of sequence is it? Describe your answer.

Row 1:	$1 + 1 = 2$
Row 2:	$1 + 2 + 1 = 4$
Row 3:	$1 + 3 + 3 + 1 = 8$
Row 4:	$1 + 4 + 6 + 4 + 1 = 16$, etc.

GOOD

The resulting numbers form a geometric progression of the form ar^{n-1}, where $a = 2$ and $r = 2$. Hence $T_n = 2(T_{n-1})$.

3/3

(e) Starting at the left hand side of Row 2, construct a diagonal going downwards to the right. The diagonal is parallel to the right hands side of Pascal's Triangle. Now start at the top of this sloping set of numbers, what is the sum of:

Its first and second numbers?	4
Its second and third numbers?	9
Its third and fourth numbers?	16
Its fourth and fifth numbers?	25

Very Good

These sums form part of a sequence of squares, ie. 4, 9, 16, 25, 36, 49, ...
where $T_n = n^2$. ✓ ✓

4/4

BIBLIOGRAPHY

Bergamini, D. (1980). *Mathematics*. Virigina: Time-Life Books, 92-93, 144-145.

Bridgwater, B. (ed.) (1969). "Leonardo Fibonacci". In *The Illustrated Columbia Encyclopedia*. New York: Columbia University Press, vol. 8, 2224.

Bridgwater, B. (ed.) (1969). "Blaise Pascal". In *The Illustrated Columbia Encyclopedia*. New York: Columbia University Press, vol. 16, 4717.

Eves, H.W. (1975). "Mathematics". In *The World Book Encyclopedia*. Chicago: Field Enterprise Educational, vol. 13, 238-244.

Goodman, H. et. al. (1994). *Future Maths*. Sydney: McGraw-Hill Book Company, 51-66, 120-128.

Jones, P.S. (1975). "Blaise Pascal". In *The World Book Encyclopedia*. Chicago: Field Enterprises Educational, vol. 15, 166.

Good Assignment
Luke
Well Done.