MATHEMATICS C

ASSIGNMENT 1

SEMESTER 1

SAINT BRENDANS' COLLEGE, YEPPOON

TOPIC: Number Systems

Due Date: 2nd June 1995 / 1st week Term 3

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ASSESSMENT CRITERION

(C1) Communication Skills . H

(C2) Mathematical Techniques A-E A 22½/25

Good effor

Title page required next time .

MATHS C SEM I ASSIGNMENT

Task One - Complete the operations table.

*	f_1	f_2	f_3	f_4	f_5	f_6
f_1	f_1	f_2	f_3	f_4	f_5	f_6
f_2	f_2	f_1	f_6	f_5	f_4	f_3
f_3	f_3	f_4	f_1	f_2	f_6	f_5
f_4	f_4	f_3	f_5	f_6	f_2	f_1
f_5	f_5	f_6	f_4	f_3	f_1	f_2
f_6	f_6	f_5	f_2	f_1	f_3	f_4

← Given example

$f_2 * f_3 = f_6$

$f_2 \circ f_3 = f_6$

$f_2[f_3(x)] = f_6(x)$ ✓

Working:-

$$f_1[f_1(x)] = f_1(x) = x = f_1(x)$$

$$f_1[f_2(x)] = f_1\left(\frac{1}{x}\right) = \frac{1}{x} = f_2(x)$$

$$f_1[f_3(x)] = f_1(1-x) = 1-x = f_3(x)$$

$$f_1[f_4(x)] = f_1\left(\frac{x-1}{x}\right) = \frac{x-1}{x} = f_4(x)$$

$$f_1[f_5(x)] = f_1\left(\frac{x}{x-1}\right) = \frac{x}{x-1} = f_5(x)$$

$$f_1[f_6(x)] = f_1\left(\frac{1}{1-x}\right) = \frac{1}{1-x} = f_6(x)$$

$$f_2[f_1(x)] = f_2(x) = \frac{1}{x} = f_2(x)$$

$$f_2[f_2(x)] = f_2\left(\frac{1}{x}\right) = \frac{1}{1/x} = x = f_1(x)$$

$$f_2[f_3(x)] = f_2\left(\frac{1}{1-x}\right) = \frac{1}{1-x} = f_6(x) \text{ Given.}$$

$$f_2[f_4(x)] = f_2\left(\frac{x-1}{x}\right) = \frac{1}{x-1/x} = \frac{x}{x-1} = f_5(x)$$

$$f_2[f_5(x)] = f_2\left(\frac{x}{x-1}\right) = \frac{1}{x/x-1} = \frac{x-1}{x} = f_4(x)$$

$$f_2[f_6(x)] = f_2\left(\frac{1}{1-x}\right) = \frac{1}{\left(\frac{1}{1-x}\right)} = 1-x = f_3(x)$$

$$f_3[f_1(x)] = f_3(x) = 1-x = f_3(x)$$

$$f_3[f_2(x)] = f_3\left(\frac{1}{x}\right) = 1-\frac{1}{x} = \frac{x-1}{x} = f_4(x)$$

$$f_3[f_3(x)] = f_3(1-x) = 1-(1-x) = x = f_1(x)$$ ✓

$$f_3[f_4(x)] = f_3\left(\frac{x-1}{x}\right) = 1-\left(\frac{x-1}{x}\right) = \frac{1}{x} = f_2(x)$$

$$f_3[f_5(x)] = f_3\left(\frac{x}{x-1}\right) = 1-\left(\frac{x}{x-1}\right) = \frac{1}{1-x} = f_6(x)$$

$$f_3[f_6(x)] = f_3\left(\frac{1}{1-x}\right) = 1-\left(\frac{1}{1-x}\right) = \frac{x}{x-1} = f_5(x)$$

$$f_4[f_1(x)] = f_4(x) = \frac{x-1}{x} = f_4(x)$$

$$f_4[f_2(x)] = f_4\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}} = 1-x = f_3(x)$$

$$f_4[f_3(x)] = f_4(1-x) = \frac{(1-x)-1}{1-x} = \frac{x}{x-1} = f_5(x)$$

$$f_4[f_4(x)] = f_4\left(\frac{x-1}{x}\right) = \frac{\left(\frac{x-1}{x}\right)-1}{\frac{x-1}{x}} = \frac{1}{1-x} = f_6(x)$$

$$f_4[f_5(x)] = f_4\left(\frac{x}{x-1}\right) = \frac{\left(\frac{x}{x-1}\right)-1}{\frac{x}{x-1}} = \frac{1}{x} = f_2(x)$$

$$f_4[f_6(x)] = f_4\left(\frac{1}{1-x}\right) = \frac{\left(\frac{1}{1-x}\right)-1}{\frac{1}{1-x}} = x = f_1(x)$$

$$f_5[f_1(x)] = f_5(x) = \frac{x}{x-1} = f_5(x)$$

$$f_5[f_2(x)] = f_5\left(\frac{1}{x}\right) = \frac{\frac{1}{x}}{\left(\frac{1}{x}\right)-1} = \frac{1}{1-x} = f_6(x)$$

$$f_5[f_3(x)] = f_5(1-x) = \frac{1-x}{(1-x)-1} = \frac{x-1}{x} = f_4(x)$$

$$f_5[f_4(x)] = f_5\left(\frac{x-1}{x}\right) = \frac{\frac{x-1}{x}}{\left(\frac{x-1}{x}\right)-1} = 1-x = f_3(x)$$

$$f_5[f_5(x)] = f_5\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1}}{\left(\frac{x}{x-1}\right)-1} = x = f_1(x)$$

$$f_5[f_6(x)] = f_5\left(\frac{1}{1-x}\right) = \frac{\frac{1}{1-x}}{\left(\frac{1}{1-x}\right)-1} = \frac{1}{x} = f_2(x)$$

$$f_6[f_1(x)] = f_6(x) = \frac{1}{1-x} = f_6(x)$$

$$f_6[f_2(x)] = f_6\left(\frac{1}{x}\right) = \frac{1}{1-\frac{1}{x}} = \frac{x}{x-1} = f_5(x)$$

$$f_6[f_3(x)] = f_6(1-x) = \frac{1}{1-(1-x)} = \frac{1}{x} = f_2(x)$$

$f_6[f_4(x)] = f_6\left(\frac{x-1}{x}\right)$
$= \frac{1}{1-\left(\frac{x-1}{x}\right)}$
$= x$
$= f_1(x)$

$f_6[f_5(x)] = f_6\left(\frac{x}{x-1}\right)$
$= \frac{1}{1-\left(\frac{x}{x-1}\right)}$
$= 1-x$
$= f_3(x)$

$f_6[f_6(x)] = f_6\left(\frac{1}{1-x}\right)$
$= \frac{1}{1-\left(\frac{1}{1-x}\right)}$
$= \frac{x-1}{x}$
$= f_4(x)$

In some cases, I found that substituting a number (eg 3) for x, I was able to simplify the function's equation faster (esp. when using a calculator).

For example, $f_5[f_4(x)] = f_5\left(\frac{x-1}{x}\right)$
$= \frac{\frac{x-1}{x}}{\left(\frac{x-1}{x}\right)-1}$

(Substitute 3 for x) $= \frac{\frac{3-1}{3}}{\left(\frac{3-1}{3}\right)-1}$
$= \frac{\frac{2}{3}}{\frac{2}{3}-1}$
$= -2$
$= 1-3$
$= 1-x$
$= f_3(x)$

The identity element is f_1 under the operation $\circ$.

IDENTITY LAW $\quad a * I = a \qquad a, I \in G$

$f_1 \circ f_1 = f_1 \qquad f_1$
$f_2 \circ f_1 = f_2$
$f_3 \circ f_1 = f_3$
$f_4 \circ f_1 = f_4$
$f_5 \circ f_1 = f_5$
$f_6 \circ f_1 = f_6$

Identity element

4/4

1/1

Task One - What is the inverse of f_5?

The inverse of f_5 is itself, f_5. ✓

Working:- INVERSE LAW $a * a^{-1} = I$ $a, a^{-1}, I \in G$

$f_5 \circ f_5 = f_1$ $\underline{f_5}$

Inverse. ✓

1/1

Task Two - Complete the group operations table.

*	f_1	f_4	f_6
f_1	f_1	f_4	f_6
f_4	f_4	f_6	f_1
f_6	f_6	f_1	f_4

Is this a group ?? $1\frac{1}{2}/2$

Working:- I took the solutions from the original table in Task One.

Task Two - What is the identity element?
The identity element is f_1, ✓ under the operation $\circ$.

Working:- As well as the proof from Task One:-
IDENTITY LAW $a * \underline{I} = a$ $a, I \in G$

$f_1 \circ \boxed{f_1} = f_1$ $\underline{f_1}$ ✓

$f_2 \circ \boxed{f_1} = f_2$

$f_3 \circ \boxed{f_1} = f_3$

Identity element

1/1

Task Two - Name the inverse of $f_1 = f_1$ ✓

$f_4 = f_4$ ✗

$f_6 = f_6$ ✗

Working:- As well as the proof from Task One:-
INVERSE LAW $a * a^{-1} = I$ $a, a^{-1}, I \in G$ ✓

$f_1 \circ \boxed{f_1} = f_1$ f_1

$f_4 \circ \boxed{f_4} = f_1$ f_4 ✗

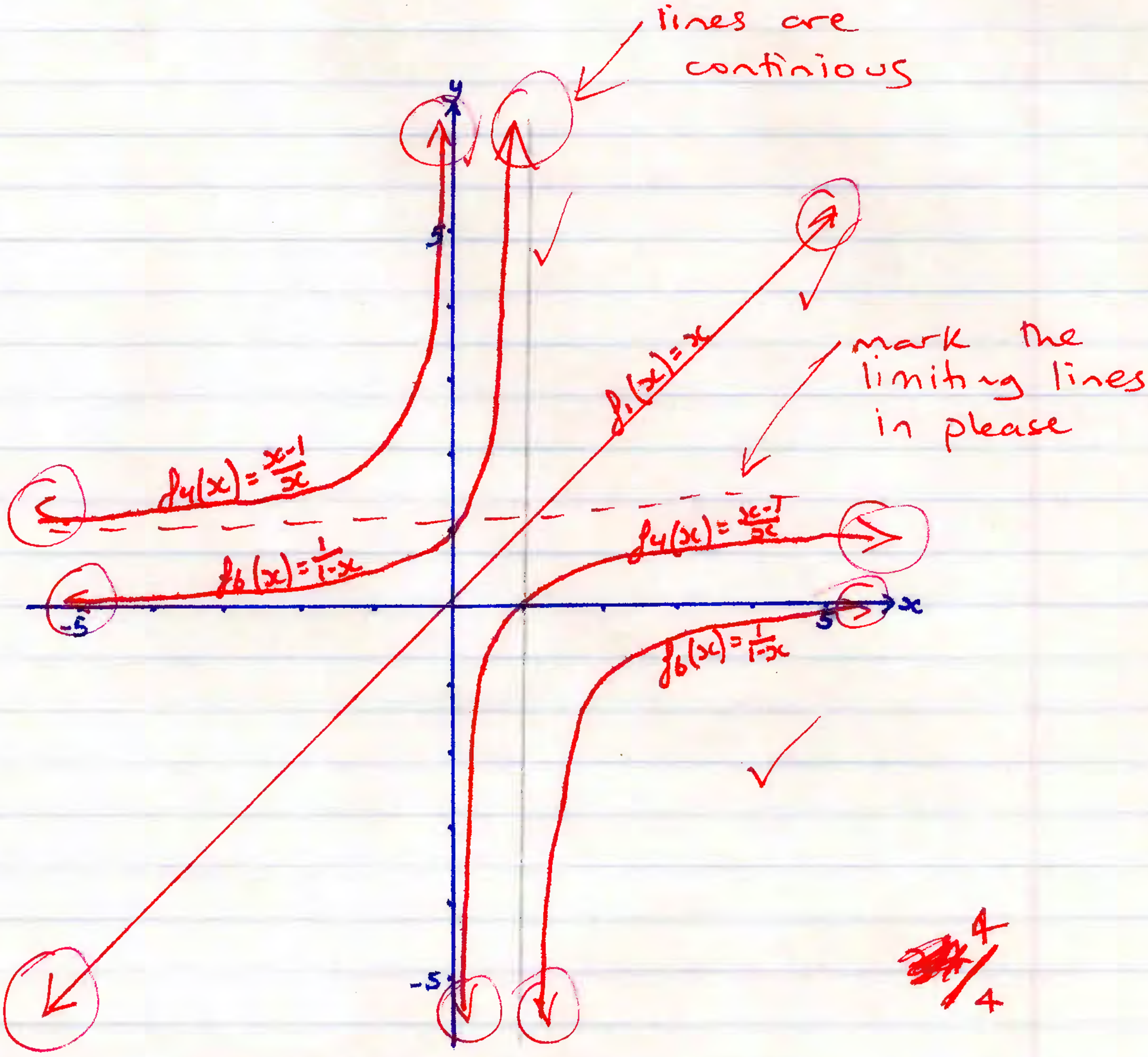
$f_6 \circ \boxed{f_6} = f_1$ f_6 ✗

Inverses

1/2

Task Three - Graph f_1, f_4 and f_6.

Plot co-ordinates on graph.



$\frac{4}{4}$

$f_1(x) = x$ Straight line.

$f_4(x) = \frac{x-1}{x}$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	1.2	1.25	$1.\dot{3}$	1.5	2	n/a	0	0.5	$0.\dot{6}$	0.7	0.8

$f_6(x) = \frac{1}{1-x}$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	$0.1\dot{6}$	0.2	0.25	$0.\dot{3}$	0.5	1	n/a	-1	-0.5	$-0.\dot{3}$	-0.25

N.B. It was necessary to plot extra points such as $x = \frac{1}{2}$, $x = \frac{1}{4}$, etc.

Task Three - Describe the inverse relationship for this group in geometric terms.

The function, $f_1(x)$, is a straight line when graphed. The function, when x is positive is inverse (reversed) to the function when x is negative and vice versa. ✓

The function, $f_4(x)$, when x is positive, runs alongside but does not actually cross the y-axis. When x is negative, it also runs alongside but does not actually cross the y-axis. $(0 < x < 0)$. The function, when x is positive is inverse to the function when x is negative and vice versa. ✗

The function, $f_6(x)$, when x is positive, runs alongside but does not actually cross $x = 1$. When x is negative, it also runs alongside but does not actually cross $x = 1$. $(1 < x < 1)$. The function, when x is positive is inverse to the function when x is negative and vice versa. ✗

Incorrect as Inverse of f_4 is f_6

f_6 is f_4

$\frac{1}{2}/1$

Task Four - Which of the following algebraic systems are groups?

(i) Whole numbers under addition

To form a group, the set of whole numbers $\{\ldots -2, -1, 0, 1, 2 \ldots\}$ under + must satisfy the following four laws:-

NOT WHOLE NUMBERS.

CLOSURE $\quad a * b = c \quad a, b, c \in G.$

$3 + 6 = 9$ True. $(3, 6, 9 \in G.)$

$(-1) + 5 = 4$ True. $(-1, 5, 4 \in G)$

$(-5) + (-8) = -13$ True. $(-5, -8, -13 \in G.)$

The closure law applies.

ASSOCIATIVITY $\quad (a*b)*c = a*(b*c) \quad a, b, c \in G$

$(3+6)+4 = 3+(6+4)$

$9 + 4 = 3 + 10$

$13 = 13$ True.

$(-2+8)+5 = -2+(8+5)$

$6 + 5 = -2 + 13$

$11 = 11$ True.

$(-7+-4)+(-3) = -7+(-4+-3)$

$-11+(-3) = -7+(-7)$

$-14 = -14$ True.

The associative law applies.

IDENTITY $\quad a * \boxed{I} = a \quad a, I \in G$

$0 + \boxed{0} = 0$

$3 + \boxed{0} = \not{0}$

$-5 + \boxed{0} = -5$

The identity element (I) is 0.

The identity law applies.

INVERSE $\quad a * \boxed{a^{-1}} = I \quad a, a^{-1}, I \in G$

$0 + \boxed{0} = 0$

$3 + \boxed{(-3)} = 0$

$-5 + \boxed{5} = 0$

The inverse elements.

The inverse law applies. 1

As all four laws apply, the set of whole numbers form a group under addition. × NO! as negative numbers are not included.

(ii) Rational numbers under addition

To form a group, the set of rational numbers, under addition, must satisfy the following four laws:- ✓

CLOSURE

$a * b = c$ ✓ $a, b, c \in G$

$2/3 + 4/7 = 1\,5/21$ True. ✓ $2/3,\ 4/7,\ 1\,5/21 \in G$

$0.475 + 6.218 = 6.693$ True. $0.475,\ 6.218,\ 6.693 \in G$

$(-6) + 39/21 = -5\,6/7$ True. $-6,\ 39/21,\ -5\,6/7 \in G$.

The closure law applies.

ASSOCIATIVITY

$(a*b)*c = a*(b*c)$ $a, b, c \in G$

$(2/3 + 4/7) + 6/10 = 2/3 + (4/7 + 6/10)$

$26/21 + 6/10 = 2/3 + 41/35$

$193/105 = 193/105$ True.

$(0.475 + 6.218) + (-2.3) = 0.475 + (6.218 + (-2.3))$

$6.693 + (-2.3) = 0.475 + 3.918$

$4.393 = 4.393$ True.

$((-6) + 39/21) + 2.7 = (-6) + (39/21 + 2.7)$

$(-4.1428572) + 2.7 = (-6) + 4.5571428$

$(-1.4428572) = (-1.4428572)$ True.

The associative law applies.

IDENTITY

$a * \boxed{I} = a$ $a, I \in G$

$2/3 + \boxed{0} = 2/3$

$0.475 + \boxed{0} = 0.475$

$(-6) + \boxed{0} = (-6)$

The identity element (I) is 0.

The identity law applies.

You should define Rational Numbers

INVERSE $\quad a * \boxed{a^{-1}} = I \qquad a, a^{-1}, I \in G$

$$\tfrac{2}{3} + \boxed{(-\tfrac{2}{3})} = 0$$
$$0{\cdot}475 + \boxed{(-0{\cdot}475)} = 0$$
$$(-6) + \boxed{6} = 0$$

The inverse elements.

The inverse law applies. 1½

As all four laws apply, the set of rational numbers form a group under addition.

GOOD

(iii) Integers under multiplication

To form a group, the set of integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$, under multiplication, must satisfy the following four laws:-

CLOSURE $\quad a * b = c \qquad a, b, c \in G$

$3 \times 6 = 18$ True $\quad (3, 6, 18 \in G)$

$(-1) \times 5 = (-5)$ True $\quad (-1, 5, -5 \in G)$

$(-5) \times (-8) = 40$ True $\quad (-5, -8, 40 \in G)$

The closure law applies.

ASSOCIATIVITY $\quad (a*b)*c = a*(b*c) \qquad a, b, c \in G$

$$(3 \times 6) \times 4 = 3 \times (6 \times 4)$$
$$18 \times 4 = 3 \times 24$$
$$72 = 72 \quad \text{True}$$
$$(-2 \times 8) \times 5 = -2 \times (8 \times 5)$$
$$-16 \times 5 = -2 \times 40$$
$$-80 = -80 \quad \text{True}$$
$$(-7 \times -4) \times -3 = -7 \times (-4 \times -3)$$
$$28 \times -3 = -7 \times 12$$
$$-84 = -84 \quad \text{True.}$$

The associative law applies.

IDENTITY $\quad a * \boxed{I} = a \quad a, I \in G$

$$\begin{aligned} 0 \times 1 &= 0 \\ 3 \times 1 &= 3 \\ -5 \times 1 &= -5 \end{aligned}$$

The identity element (I) is 1.
The identity law applies.

INVERSE $\quad a * \boxed{a^{-1}} = I \quad a, a^{-1}, I \in G$

$$\begin{aligned} 0 \times (1/0) &= 1 \\ 3 \times 1/3 &= 1 \\ -5 \times -1/5 &= 1 \end{aligned}$$

The inverse elements are not integers.
The inverse law does NOT apply. ✓

As the inverse law does not apply, the set of integers do not form a group under multiplication. ✓ GOOD. $1\frac{1}{2}$

(iv) Rational numbers under multiplication

To form a group, the set of rational numbers, under multiplication, must satisfy the following four laws:-

CLOSURE $\quad a * b = c \quad a, b, c \in G$

$2/3 \times 4/7 = 8/21$ True ($2/3, 4/7, 8/21 \in G$)

$0.475 \times 6.218 = 2.95355$ True ($0.475, 6.218, 2.95355 \in G$)

$-6 \times 39/21 = -11.142856$ True ($-6, 39/21, -11.142856 \in G$)

The closure law applies.

ASSOCIATIVITY $\quad (a*b)*c = a*(b*c) \quad a, b, c \in G$

$$\begin{aligned} (2/3 \times 4/7) \times 6/10 &= 2/3 \times (4/7 \times 6/10) \\ 8/21 \times 6/10 &= 2/3 \times 12/35 \\ 24/105 &= 24/105 \quad \text{True.} \end{aligned}$$

$$\begin{aligned} (0.475 \times 6.218) \times -2.3 &= 0.475 \times (6.218 \times -2.3) \\ 2.95355 \times -2.3 &= 0.475 \times -14.3014 \\ -6.793165 &= -6.793165 \quad \text{True.} \end{aligned}$$

$$(-6 \times {}^{39}/_{21}) \times 2.7 = -6 \times ({}^{39}/_{21} \times 2.7)$$

$$-11.142856 \times 2.7 = -6 \times 5.0142855$$

$$-30.085711 = -30.085713 \quad \text{True.}$$

The associative law applies.

IDENTITY $\qquad a * \boxed{I} = a \qquad a, I \in G$

$${}^{2}/_{3} \times \boxed{1} = {}^{2}/_{3}$$

$$0.475 \times \boxed{1} = 0.475$$

$$-6 \times \boxed{1} = -6$$

The identity element is 1.

The identity law applies.

INVERSE $\qquad a * \boxed{a^{-1}} = I \qquad a, a^{-1}, I \in G$

$${}^{2}/_{3} \times \boxed{{}^{3}/_{2}} = 1$$

$$0.475 \times \boxed{2.1052631} = 1$$

$$-6 \times \boxed{-{}^{1}/_{6}} = 1$$

The inverse elements.

The inverse law applies.

As all four laws apply, the set of rational numbers form a group under multiplication.

What about an inverse for 0

1

Task Four - Show that $f_4 \circ (f_6 \circ f_4) = (f_4 \circ f_6) \circ f_4$

(Use orig. table from Task One.)

$$f_4 \circ f_1 = f_1 \circ f_4$$

$$f_4 = f_4$$

QED.

1

Task Four – Does the group of functions have any subgroups other than the one mentioned previously? Is so, name one.

One subgroup is of order 2:-

$\circ$	f_1	f_5
f_1	f_1	f_5
f_5	f_5	f_1

✓

Other order 2 subgroups are:-
$G_A\{f_1, f_2\}$ and $G_B\{f_1, f_3\}$.

WORKING !!
EXPLANATION

$1\frac{1}{2}/2$