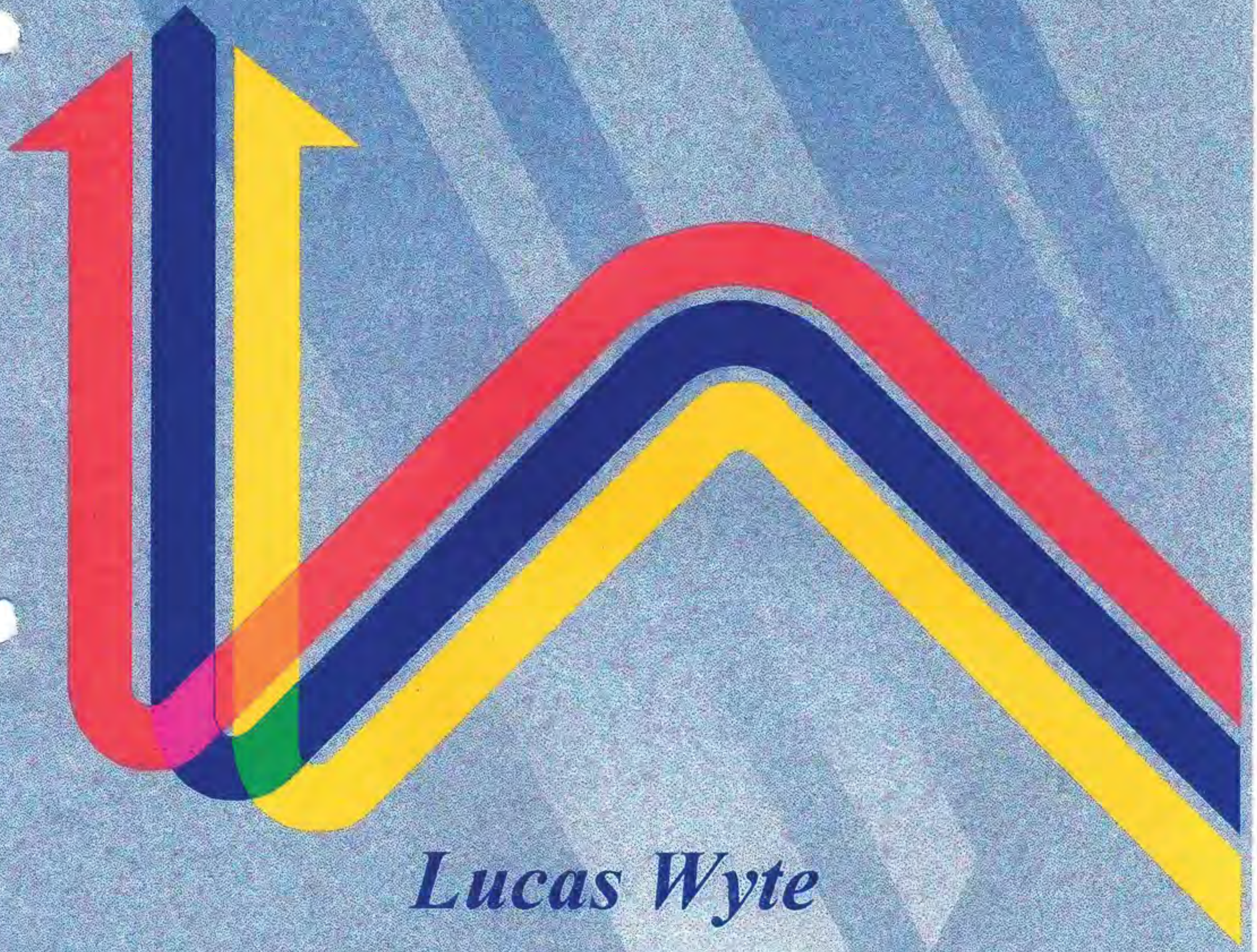
EXPONENTIAL AND TRIGONOMETRIC FUNCTIONS



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12 Mathematics C

Semester III, 1996.

12 MATHEMATICS C

Semester III Alternative Assessment

EXPONENTIAL AND TRIGONOMETRIC FUNCTIONS

General Shape

The exponential trigonometric function $y = e^{ax}\sin bx$ is a product of two functions, being: $y = e^{ax}$ and $y = \sin bx$, both of which were introduced previously in Mathematics B.

The general shape of the equation $y = e^{ax}\sin bx$ can be seen in the graph below (fig. 1).

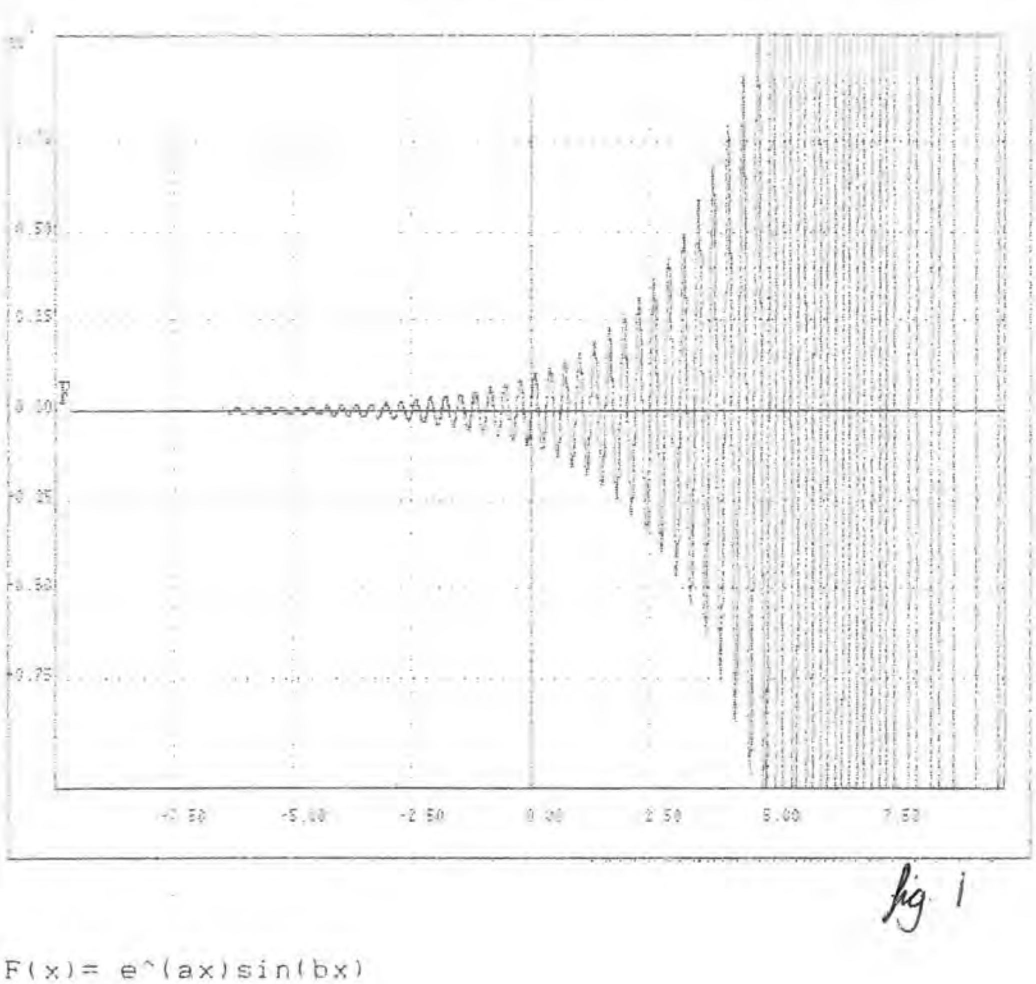

fig 1

```
F(x)= e^(ax)sin(bx)

a=      0.50000
b=     20.00000
```

y* = *a* sin *bx

From previous knowledge acquired in Mathematics B, it is known that the regular $y = a \sin bx$ graph's (fig. 2) amplitude is determined by *a*. The greater the value of *a*, the greater the amplitude. Additionally, we know that the function's wavelength and in turn its frequency, are determined by the value of *b*. The greater the value of *b*, the lesser the wavelength and in turn, the greater the frequency of wave cycles.

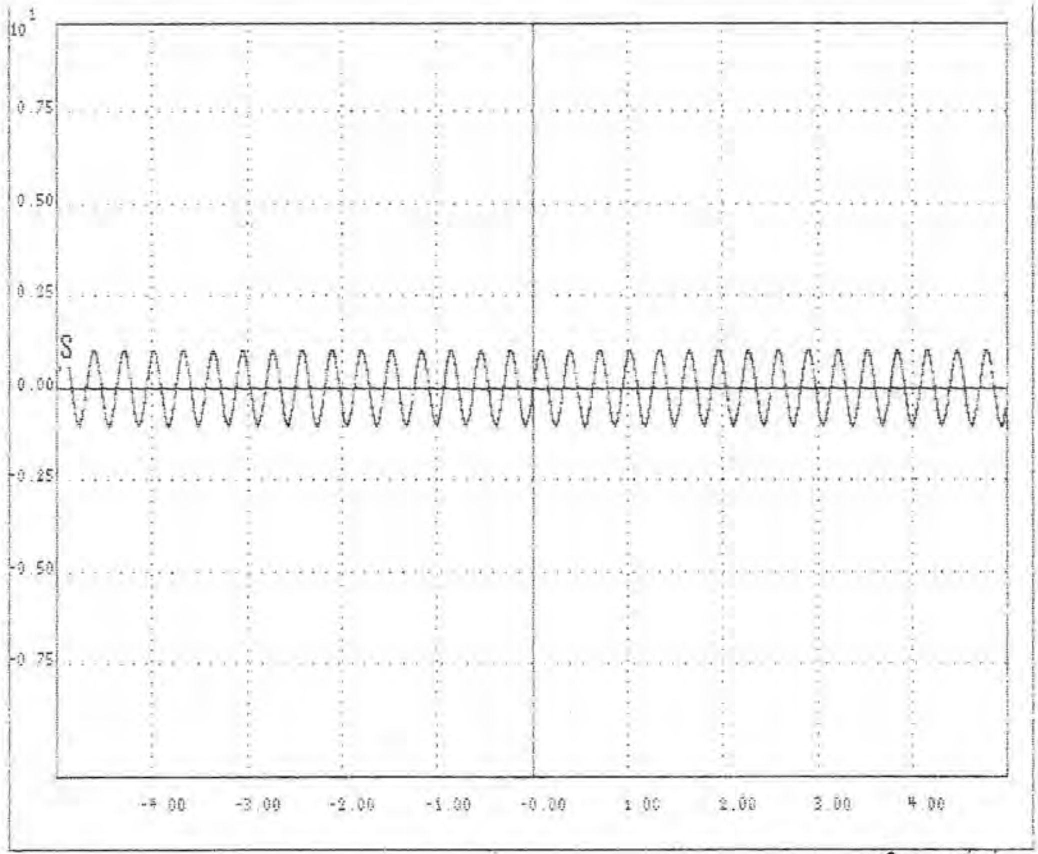

fig. 2

S(x)= sin(bx)

a= 0.50000
b= 20.00000

$y = e^{ax}$

Similarly, it is understood from previous studies in Mathematics B that the gradient of the function $y = e^{ax}$ (fig. 3) is determined by the value a, relative to the variable x. The greater the value of a (and naturally, the greater the value of x as it increases) results in a greater gradient. In addition, when this exponential function is made negative, the graph is simply "flipped" horizontally around the x-axis, as seen below.

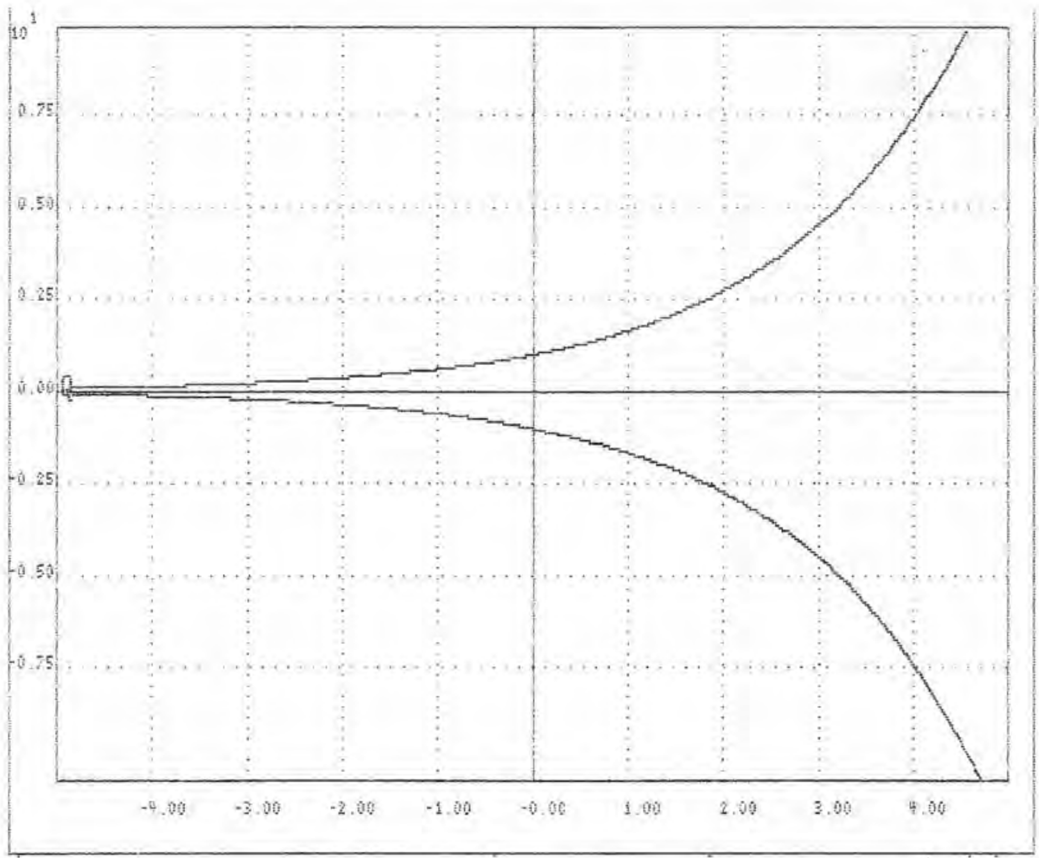

fig. 3

```
P(x)= e^(ax)
Q(x)= -e^(ax)

a=     0.50000
b=    20.00000
```

When the exponential function $y = e^{ax}$ and its negative are graphed with the exponential trigonometric function $y = e^{ax}\sin bx$ (fig. 4) it is clearly evident that e^{ax} forms the amplitude of $y = e^{ax}\sin bx$, while the wavelength and frequency of the graph behaves normally (as in $y = \sin bx$).

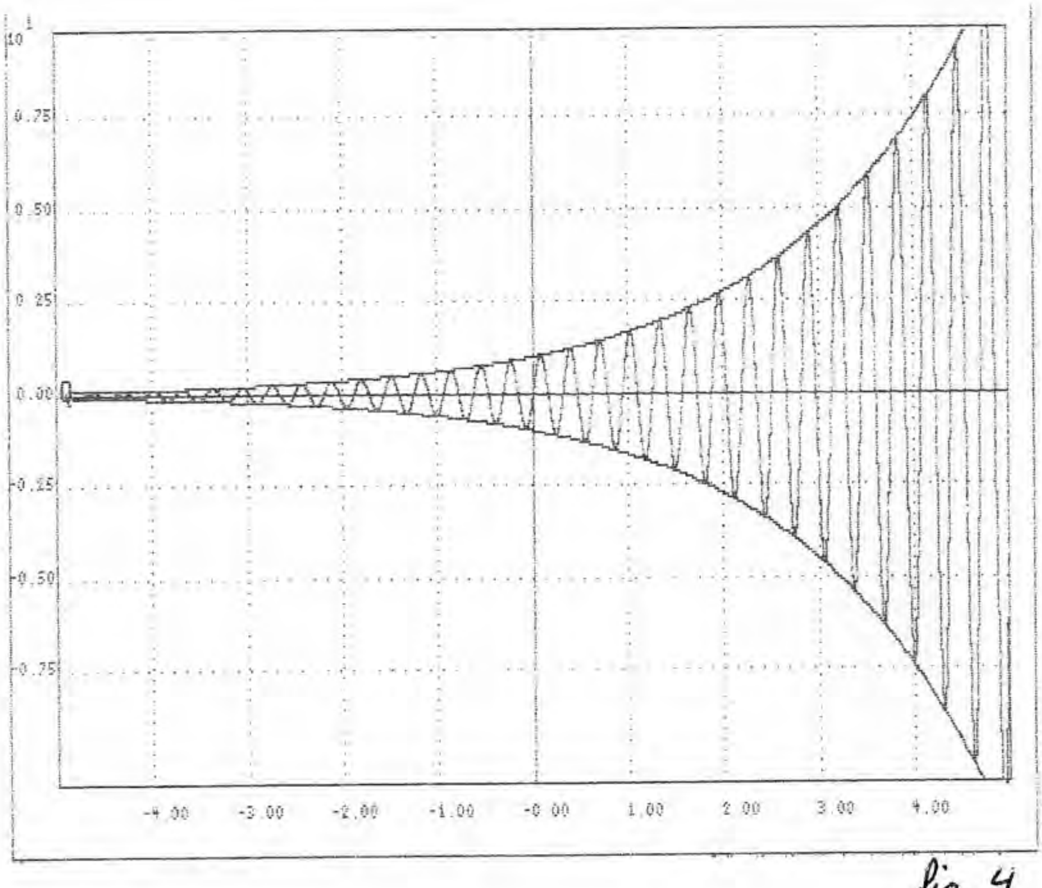

fig. 4

```
F(x)= e^(ax)sin(bx)
P(x)= e^(ax)
Q(x)= -e^(ax)

a=     0.50000
b=    20.00000
```

Investigation of Change in Values of *a* and *b*

To investigate the effect of changes to the variables *a* and *b* on the graph of $y = e^{ax}\sin bx$, we must first establish starting values for the variables. For this exercise, *a* will be equal to 0.5 and *b* will be equal to 5.0 (fig. 5).

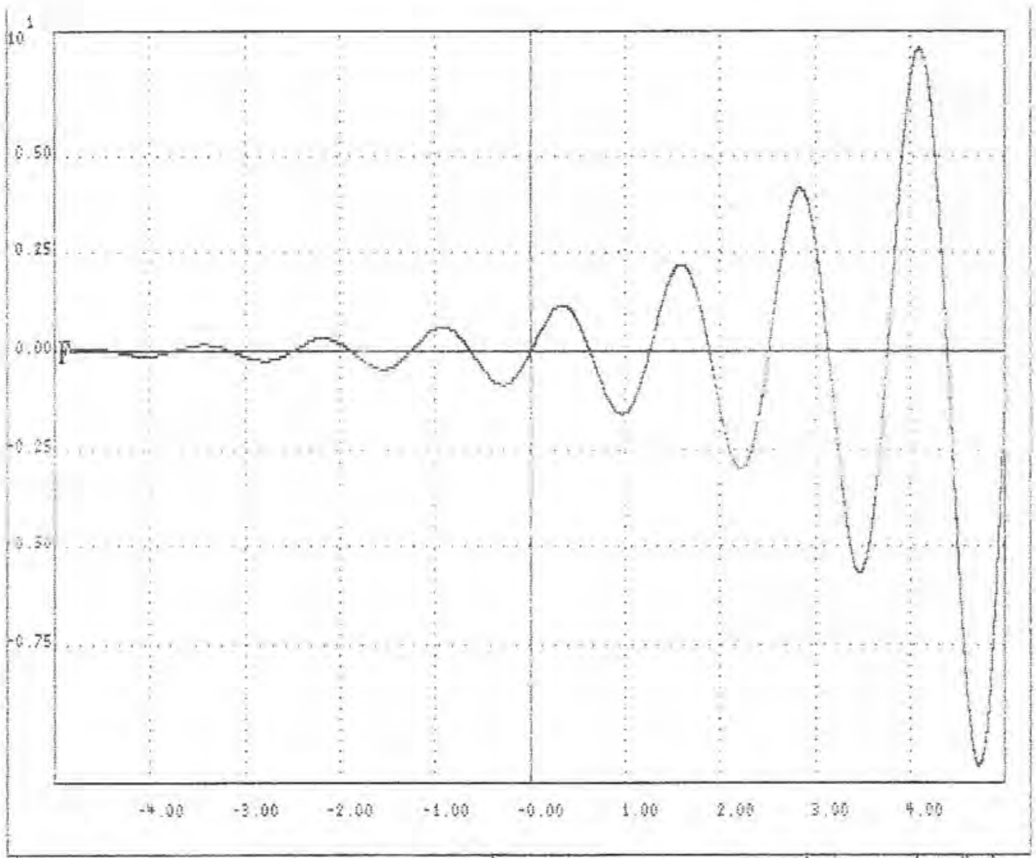

fig 5

F(x)= e^(ax)sin(bx)

a= 0.50000
b= 5.00000

Change in a - $a_n < a_m$

It can be seen that if we decrease the value of a (in this case to 0.2), the amplitude of the wave will be less (fig. 6). That is, the gradient at any point on the $y = e^{ax}$ curve is more shallow.

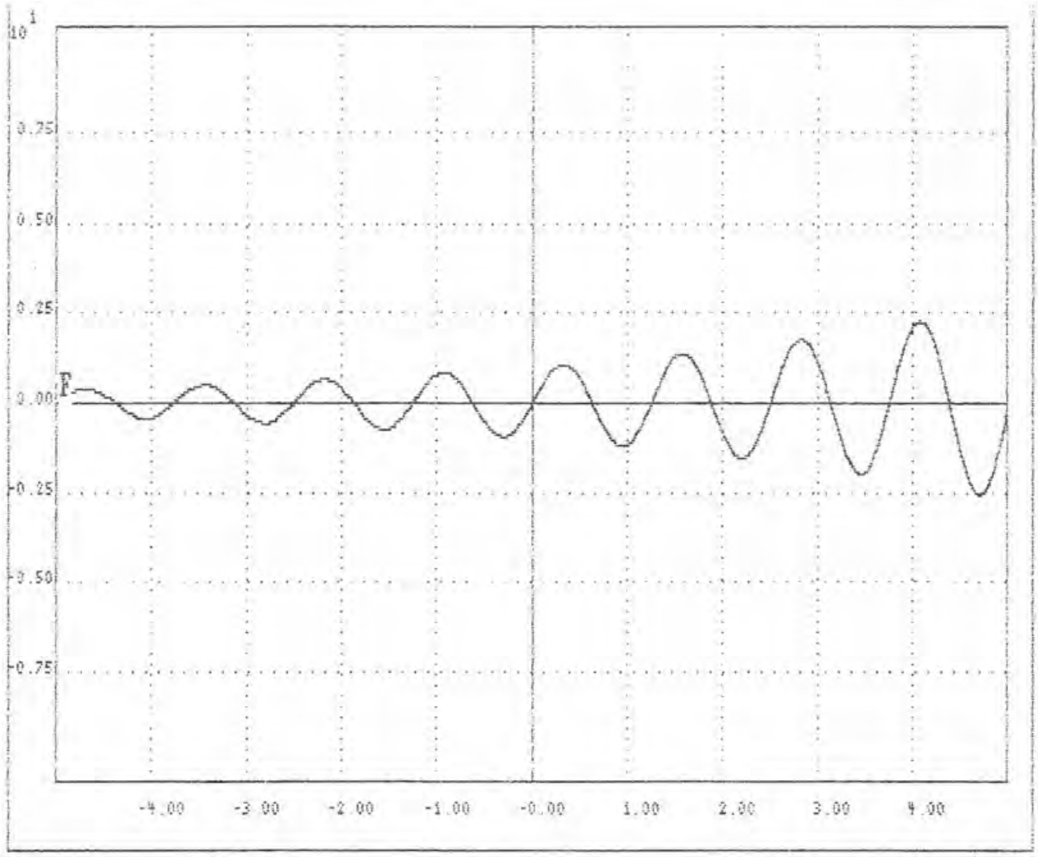

fig. 6

F(x)= e^(ax)sin(bx)

a= 0.20000
b= 5.00000

Change in a - $a_n > a_m$

Similarly, if we increase the value of a (in this case to 0.9), the amplitude of the wave will be greater (fig. 7). That is, the gradient at any point on the $y = e^{ax}$ curve is steeper.

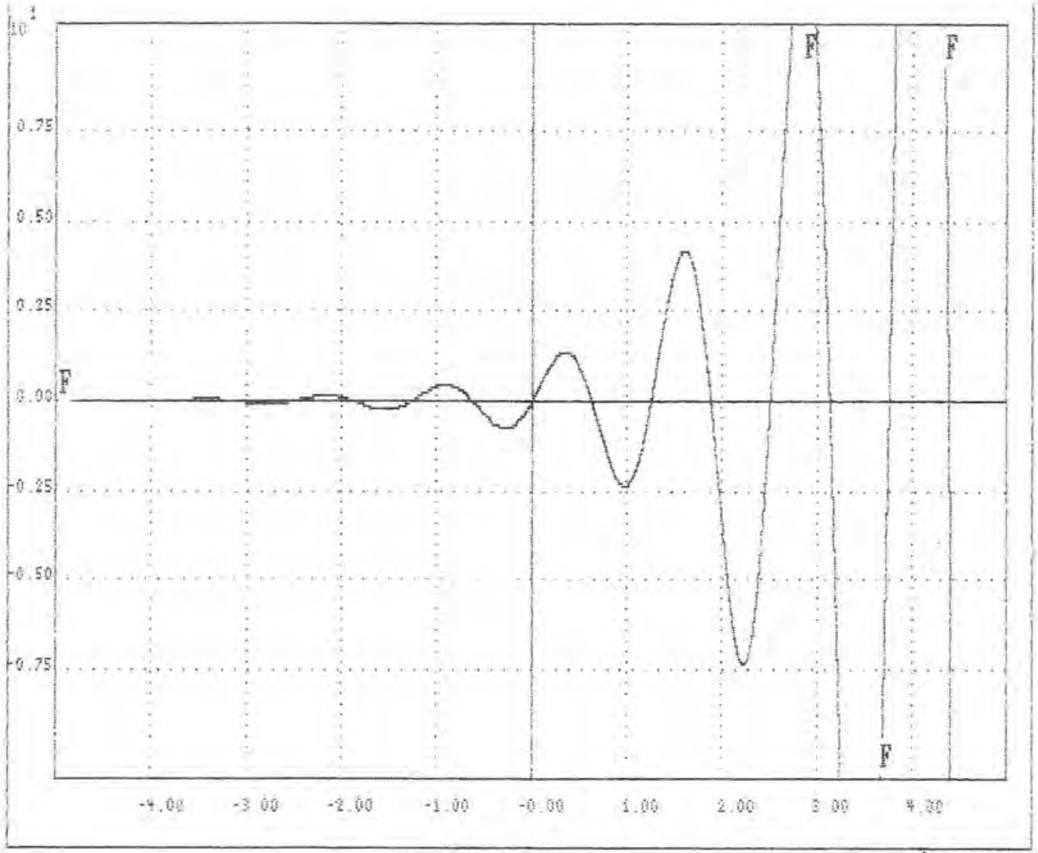

fig. 7

F(x)= e^(ax)sin(bx)

a= 0.90000
b= 5.00000

Therefore, it can be seen that the amplitude of the $y = e^{ax}\sin bx$ wave is proportional to the value of a.

Change in *a* - *-a*

Additionally, if a is given a negative value, the $y = e^{ax}\sin bx$ wave is simply "rotated" vertically around the y-axis (fig. 8). Upon closer examination of this graph, it can be seen that the amplitude of the wave is not really proportional to the value of a, but is in fact proportional to the absolute value of a.

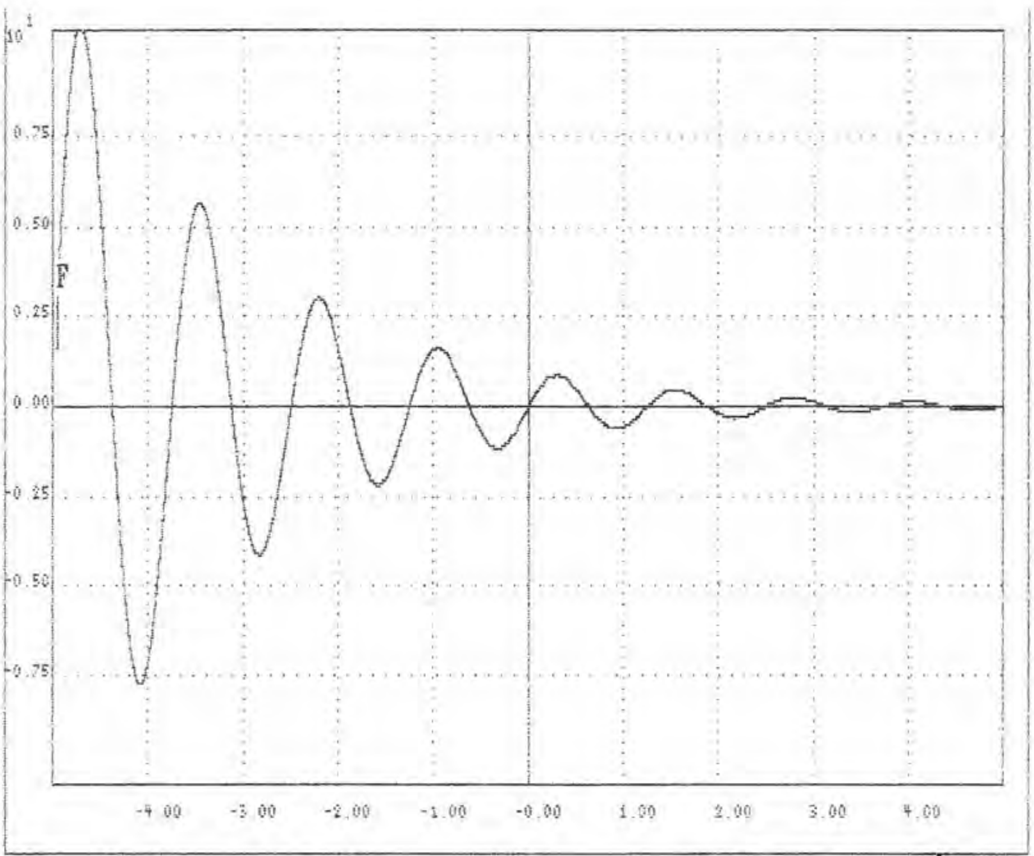

fig 8

F(x)= e^(ax)sin(bx)

a= -0.50000
b= 5.00000

Change in b - $b_n < b_m$

On the other hand, it can be seen that if we decrease the value of b (in this case to 2.0), the wavelength of the $y = e^{ax}\sin bx$ graph will be greater (fig. 9). In turn, the frequency of wave cycles is decreased (the waves are further apart). However, it should be noted that the amplitude of the graph remains the same.

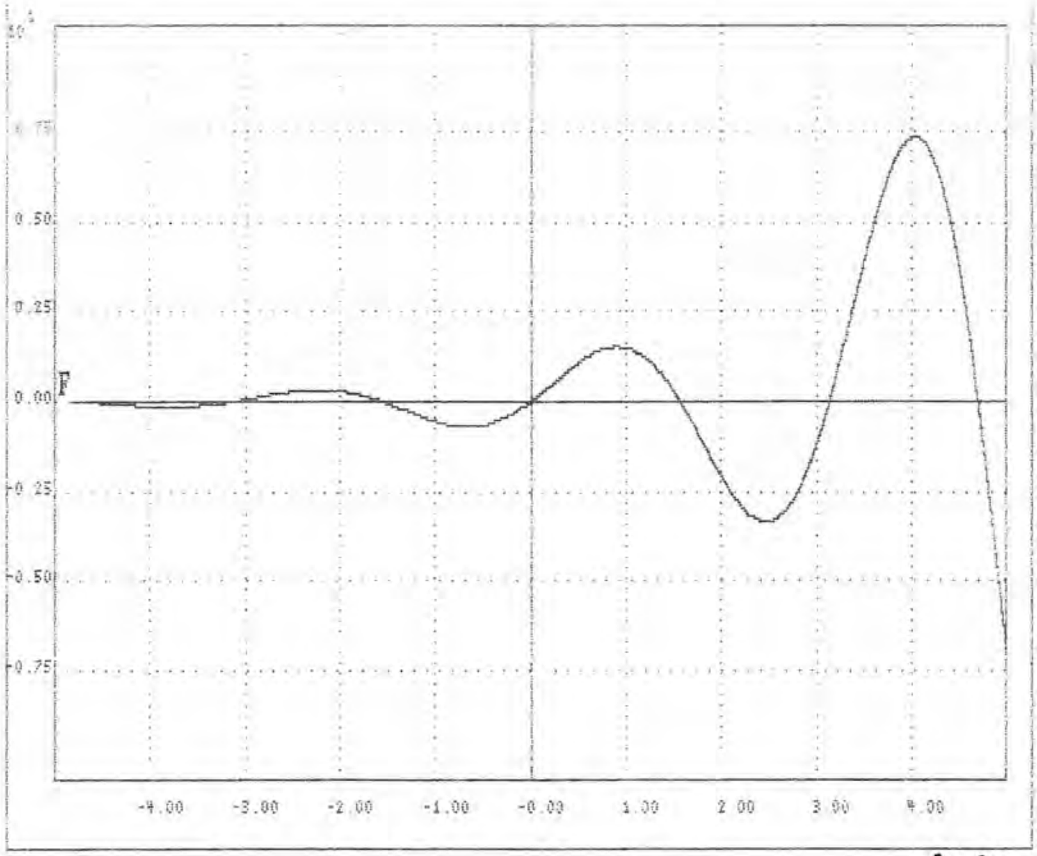

fig. 9

F(x)= e^(ax)sin(bx)

a= 0.50000
b= 2.00000

Change in b - $b_n > b_m$

Similarly, if we increase the value of b (in this case to 9.0), the wavelength of the $y = e^{ax}\sin bx$ graph will be greater (fig. 10). In turn, the frequency of wave cycles is increased (the waves are closer together). Once again, it can be seen that the amplitude of the graph remains the same.

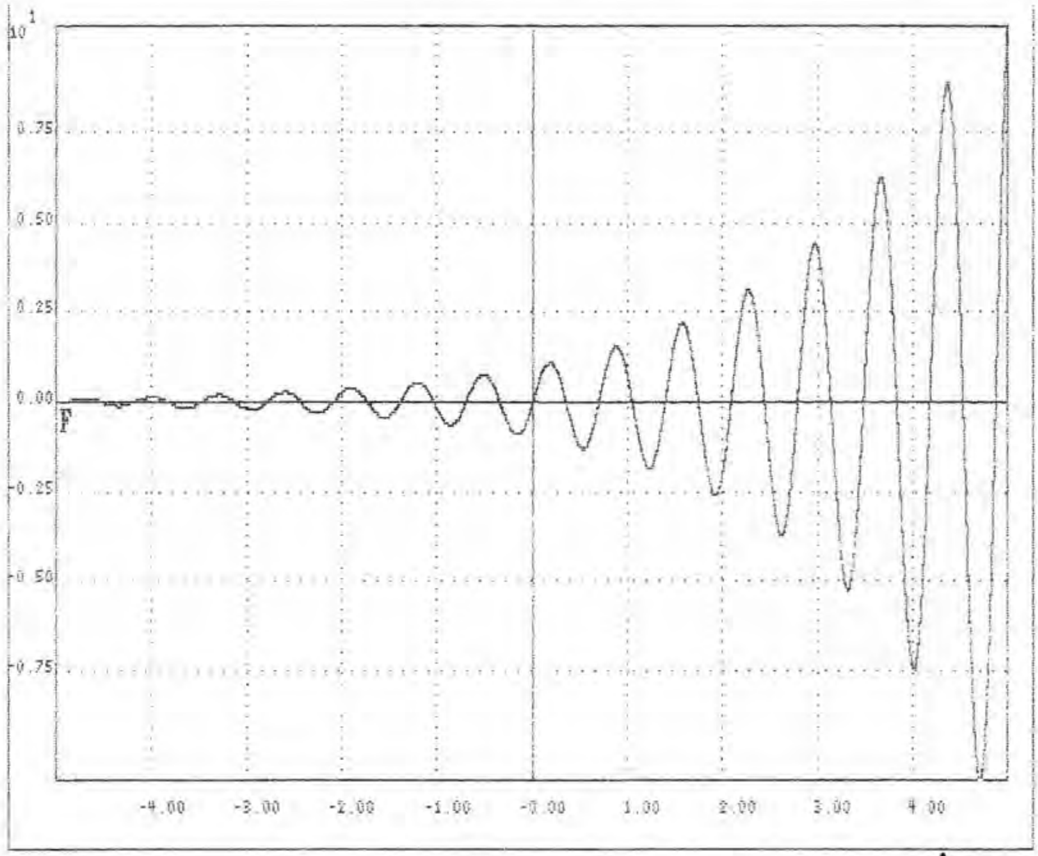

fig. 10

F(x)= e^(ax)sin(bx)

a= 0.50000
b= 9.00000

Therefore, it is evident that a change in the value of b promotes a change in wavelength of the $y = e^{ax}\sin bx$ graph and in turn, the frequency of cycles.

Change in *b* - *-b*

In addition, if *b* is given a negative value, the $y = e^{ax}\sin bx$ wave is simply "reflected" horizontally to the *x*-axis (fig. 11). This is the equivalent of making the entire function negative. In other words, $y = e^{ax}\sin(-bx) = -e^{ax}\sin(bx)$ (fig. 12).

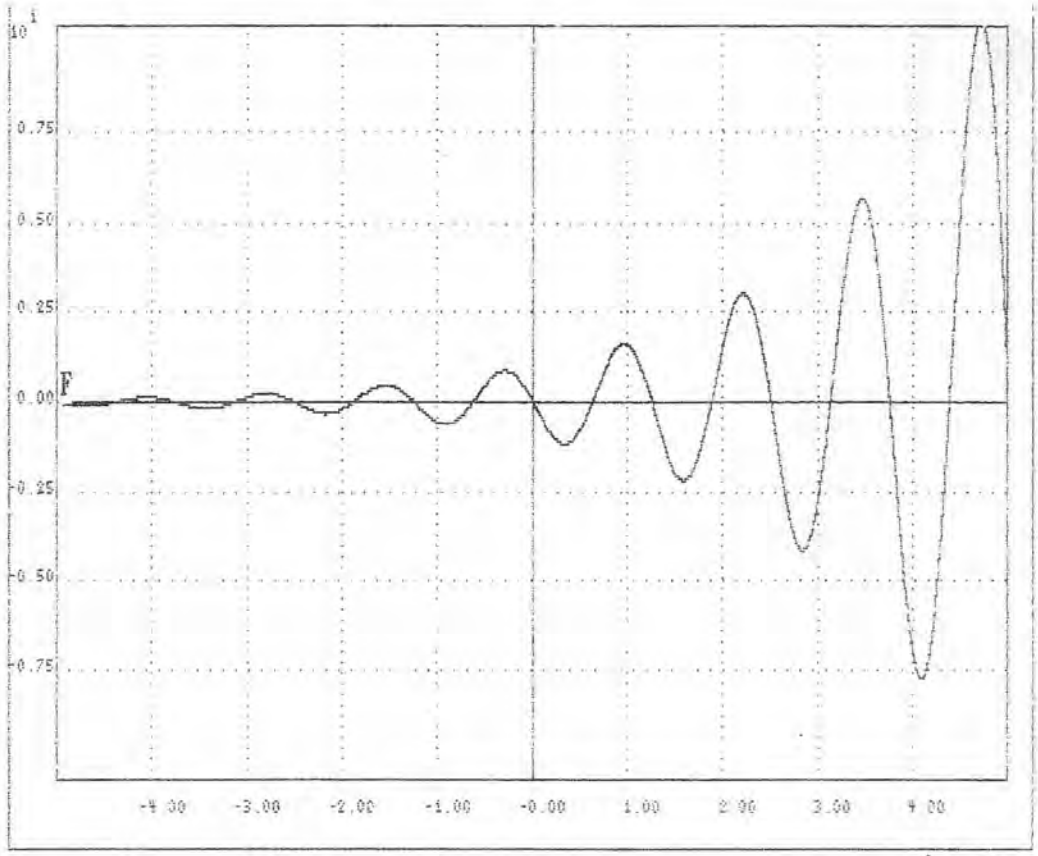

fig. 11

F(x)= e^(ax)sin(bx)

a= 0.50000
b= -5.00000

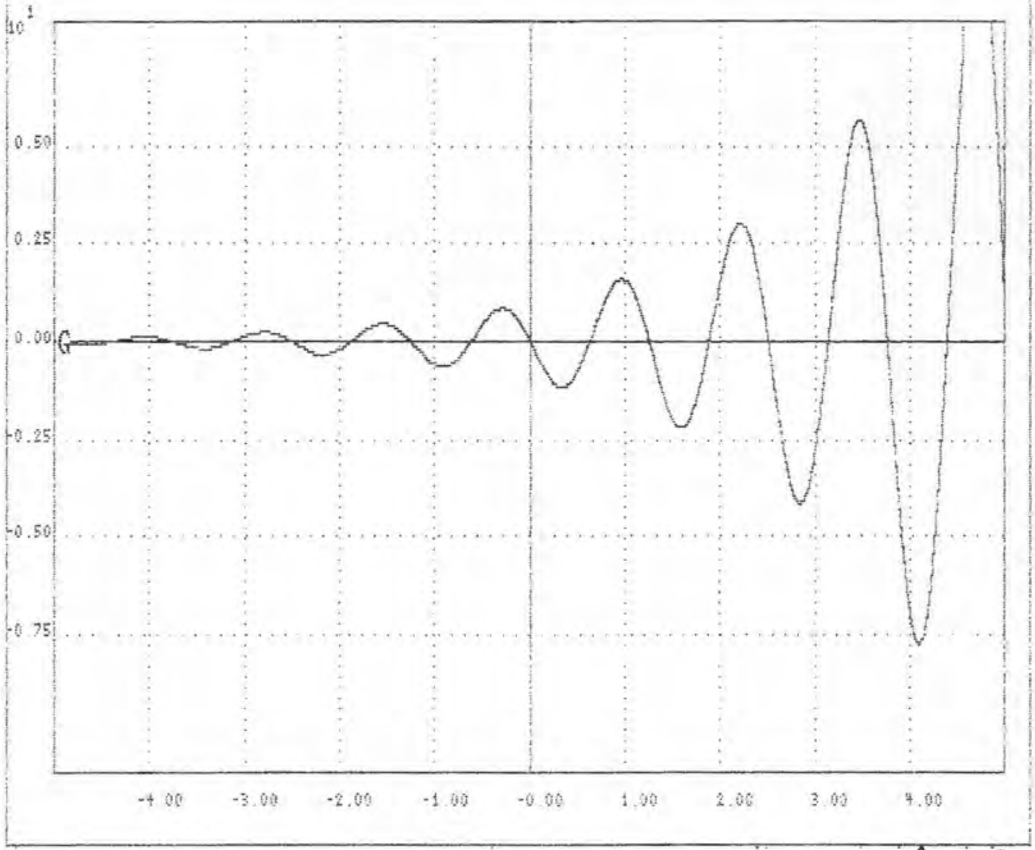

fig. 12

G(x)= -e^(ax)sin(bx)

a= 0.50000
b= 5.00000

$y = -e^{ax}\sin bx$

It should also be noted that the graph of $y = -e^{ax}\sin bx$ is the horizontally "mirrored" graph of the function $y = e^{ax}\sin bx$ (fig. 13). The amplitude remains the same in each case, as does the wavelength and frequency of cycles.

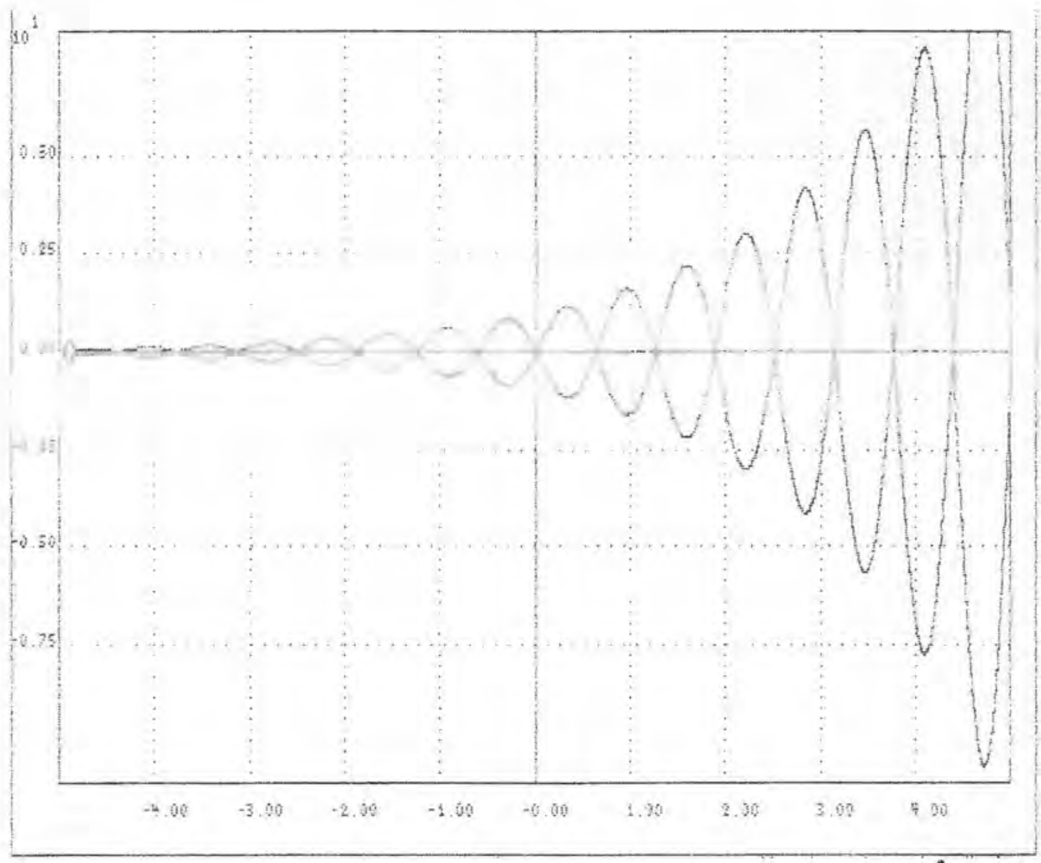

fig. 13

F(x)= e^(ax)sin(bx)
G(x)= -e^(ax)sin(bx)

a= 0.50000
b= 5.00000

Application - Bungee Jumping

Bungee jumping is a daredevil activity in which the jumper leaps headfirst off a bridge, tower, crane, or hot-air balloon from heights of about 23-91 metres with one end of a long, wrist-thick, nylon-encased (elastic) bungee cord tethered to the ankles or to a body harness, and the other end attached to the jumping-off place. After a few seconds of a free fall the cord begins to stretch, slowing the dive until the jumper is within a few meters of the surface below. Then he or she is snapped upward as the cord recoils to its original length, and continues to travel higher with the recoil's momentum. After several yo-yo-like rebounds, the jumper is lowered and disengaged.

The sport of bungee jumping can be related to the mathematics of exponential trigonometric functions. If we use the function $y = e^{ax}\sin bx + c$ (fig. 14), making the value of a negative (for a declining altitude) and c being a vertical shift in the position of the wave, we have a graphical representation of a jumper's dive. (It should be noted that our domain allows for only positive values for x.)

The variables in this real-life application are:

Let x = time (x can only be positive, ie. $x > 0$.);
Let y = height (y can only be positive, ie. $y > 0$.);
Let $|a|$ = proportional to jumper's weight (a = jumper's weight (prop) as a negative value.);
Let b = elasticity of the bungee cord. ? Explain

Therefore, the greater the jumper's weight ($|a|$), the larger the distance covered by the dive and the more elastic the bungee cord is (b), the longer the time takes for the cord to recoil.

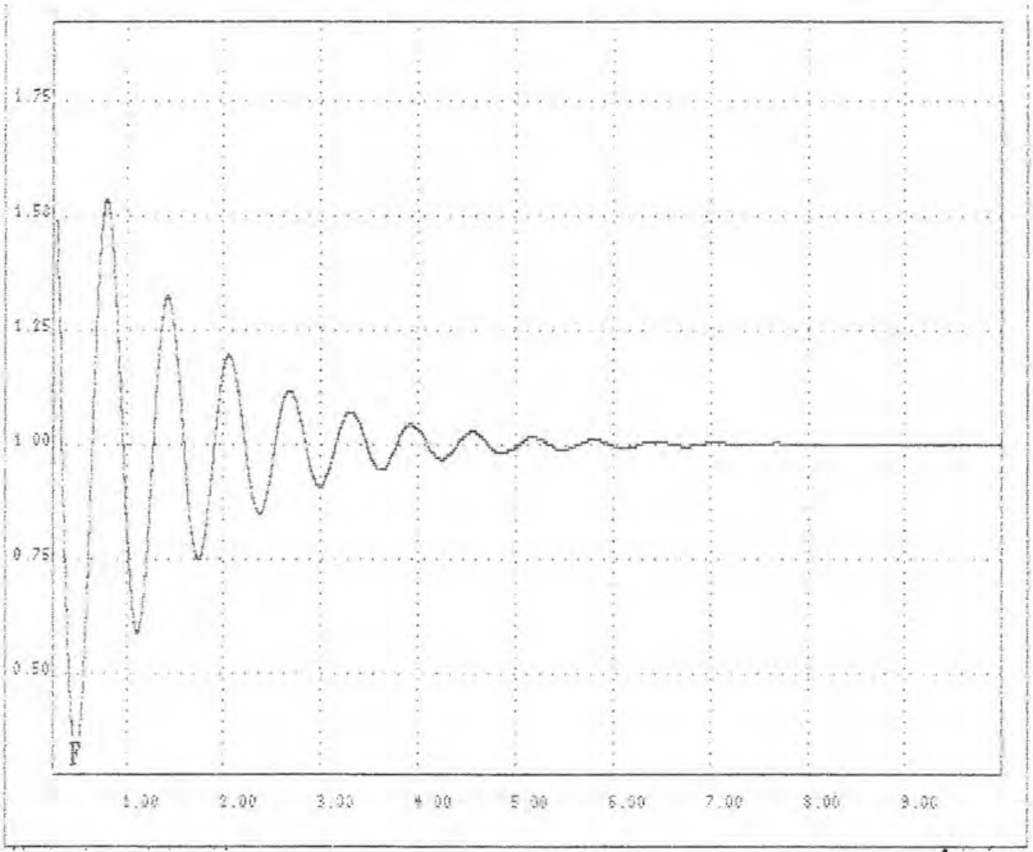

fig. 14

F(x)= e^(ax)sin(bx)+c

a= -0.80000
b= 10.00000
c= 1.00000

Application - Metronome

The metronome is a mechanical timing device used to maintain a tempo for music. An inverted pendulum powered by a spring is used to produce audible beats, ranging from 40 to 208 beats per minute, usually for the guidance of pianists. In Maelzel's design, which shall be used in this example, features an adjustable, sliding weight which fits snugly on a calibrated pendulum rod and alters the period (frequency) of the pendulum.

The metronome and its use of the pendulum can also be related to the mathematics of exponential trigonometric functions. If we take the original function $y = e^{ax} \sin bx$ (fig. 15), making the value of a negative (for a declining altitude), we have a graphical representation of a metronome's swing. (It should be noted that our domain allows for only positive values for x.)

The variables in this real-life application are:

- Let x = time (x can only be positive, ie. $x > 0$.);
- Let y = pendulum weight's position as it moves ($-y$ = left-most position, $+y$ = right-most position);
- Let $|a|$ = proportional to pendulum weight's distance from centre (a = distance (prop) as a negative value; is a constant.);
- Let b = pendulum weight's position on the pendulum rod.

Therefore, the further up the pendulum rod the weight is positioned (b), the less beats per period of time. (The distance the pendulum weight travels from the centre ($|a|$) cannot be changed and is therefore a constant in this example.)

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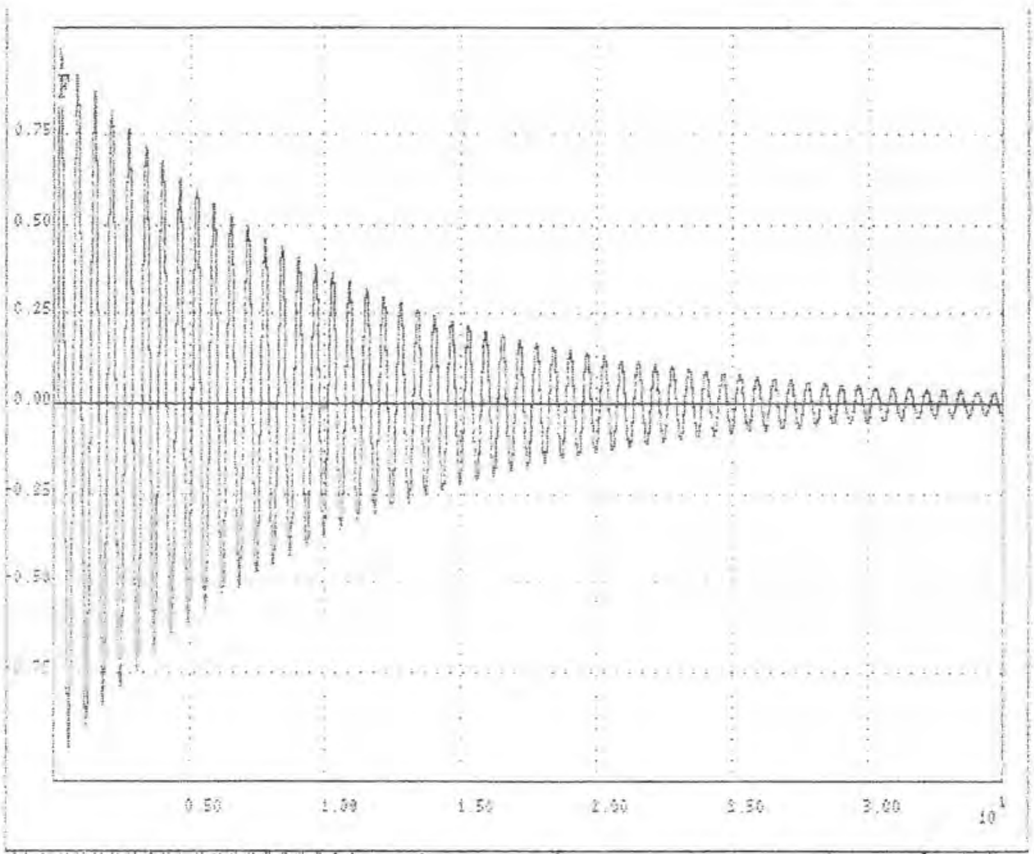

fig. 15

F(x)= e^(ax)sin(bx)

a=   -0.10000
b=   10.00000

Conclusion

The exponential trigonometric function $y = e^{ax}\sin bx$ has two variables, a and b, which determine the shape of the graph. The absolute value $|a|$ is proportional to the wave's amplitude while the value b relates to the graph's wavelength, and in turn its frequency of wave cycles.

This exponential trigonometric function can also be used as a mathematical and graphical representation of real-life applications in today's modern world, where an oscillation gradually slows to a near halt.